Noncritical quadrature squeezing through spontaneous polarization symmetry breaking

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We discuss the possibility of generating noncritical quadrature squeezing by spontaneous polarization symmetry breaking. We first consider Type II frequency-degenerate optical parametric oscillators but discard them for a number of reasons. Then we propose a four-wave-mixing cavity, in which the polarization of the output mode is always linear but has an arbitrary orientation. We show that in such a cavity, complete noise suppression in a quadrature of the output field occurs, irrespective of the parameter values. © 2010 Optical Society of America

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Spontaneous rotational symmetry breaking has recently been shown theoretically to be a resource for noncritical quadrature squeezing in both Type I frequency-degenerate optical parametric oscillators (DOPOs) [1,2] and degenerate four-wave-mixing $\chi^{(3)}$ cavities [3], as was previously shown for spontaneous translational symmetry breaking in wide-aperture Type I DOPOs [4,5]. The underlying idea is that a spontaneous symmetry breaking occurring within a nonlinear cavity entails the existence of a canonical pair of observables: one of them is solely driven by quantum fluctuations (such as the orientation of a Hermite–Gauss signal mode in [1–3] or the location in the transverse plane of a localized structure in [4,5]), while its canonical pair is maximally damped and, consequently, maximally insensitive to quantum fluctuations (angular and linear momenta, respectively, in the referred cases). We have predicted in the above cases that by this means, perfect quadrature squeezing appears in the maximally damped mode independently of the parameter setting, hence the name *noncritical* quadrature squeezing. In the present Letter, we discuss the possibility of achieving the same result through a spontaneous polarization symmetry breaking (SPSB).

To achieve noncritical quadrature squeezing through SPSB, the nonlinear cavity must verify some prerequisites. It must possess a free polarization parameter (FPP), that is, a parameter of the signal field polarization [6,7] not fixed by the system dynamic equations. Additionally, it must allow for the existence of a nonzero mean field solution for the signal field; in this way, once the threshold for the generation of the signal mode is crossed, the random occurrence of a particular FPP will automatically produce an SPSB. From a mathematical point of view, a system like this has a Goldstone mode (a mode with null eigenvalue in the stability matrix irrespective of the system parameters), reflecting the equal likeliness of any value of the FPP. Additionally, associated to this mode, a maximally damped mode emerges; this mode can be proved to be the canonical pair of the FPP, and it coincides with a quadrature of the mode crosspolarized with respect to the generated one. Hence, quantum fluctuations can freely act onto the FPP, making it completely undetermined, allowing then for the complete noise reduction of its canonical pair via the Heisenberg uncertainty principle, that is, perfect squeezing of the aforementioned quadrature. The above requirements can be met, at least in principle, in both $\chi^{(2)}$ and $\chi^{(3)}$ nonlinear cavities.

Let us first consider a $\chi^{(2)}$ cavity. Upconversion processes (such as second-harmonic generation) cannot help for our purposes, as the generated field appears linearly in the Hamiltonian and all its properties are fixed by the subharmonic modes. With respect to downconversion processes, the only possibility is a Type II optical parametric oscillator (OPO). In its usual configuration, Type II parametric downconversion takes place in two modes having orthogonal linear polarizations and an *undefined relative phase* [8]. If, in addition, the downconverted fields have the same frequency, the process just described is equivalent to the spontaneous generation of a field with an elliptical polarization along the $\pm 45°$ axes having an arbitrary eccentricity and direction of rotation (see below). Then a Type II frequency-degenerate OPO is a candidate for SPSB. In the interaction picture, the Hamiltonian of such a system can be written as

$$\hat{H}_{\mathrm{OPO}} = i\hbar(\mathcal{E}_p \hat{b}^{\dagger} + \chi \hat{b} \hat{a}_x^{\dagger} \hat{a}_y^{\dagger}) + \mathrm{H.c.}, \qquad (1)$$

where $\mathcal{E}_p$ is the pumping field amplitude; $\hat{b}^{\dagger}$, $\hat{a}_x^{\dagger}$, and $\hat{a}_y^{\dagger}$ are the creation operators for the pump mode and the $\mathbf{e}_x$ and $\mathbf{e}_y$ polarized signal modes, respectively; and χ is the nonlinear coupling constant. This Hamiltonian has the symmetry $(\hat{a}_x, \hat{a}_y) \to (\hat{a}_x e^{i\theta}, \hat{a}_y e^{-i\theta})$ that leaves undefined the phase difference between the signal modes 2θ. This is the FPP of the system, which, as explained below, is directly related to the eccentricity of the signal field polarization ellipse. Associated with this symmetry is a constant of motion, namely, the photon number difference between the signal modes $\hat{a}_x^{\dagger}\hat{a}_x - \hat{a}_y^{\dagger}\hat{a}_y$. This ensures that signal photons are created in pairs, and, hence, the two polarization modes $\mathbf{e}_x$ and $\mathbf{e}_y$ will have exactly the same properties: they are *twin beams* whose intensity difference is potentially perfectly squeezed [9,10].

The Hamiltonian Eq. (1) is isomorphic to that of [1,2]. There we analyzed squeezing generation through spontaneous rotational symmetry breaking in a Type I DOPO, in which the two signal modes had opposite orbital angular momenta (two-TM DOPO). Hence all the results found in that system apply to the current case. In particular, we demonstrated that (i) for $\mathcal{E}_p > \gamma_p \gamma_s / \chi$ ($\gamma_{p/s}$ is the cavity damping rate at the pump/signal frequency) a steady, nonzero mean field appears at the signal frequency in the mode $\mathbf{e}_b = (\mathbf{e}_x e^{-i\theta} + \mathbf{e}_y e^{i\theta})/\sqrt{2}$, an elliptically polarized mode as explained above whose eccentricity depends on θ [7] (*bright mode* in the following, as it is macroscopically occupied); (ii) starting from a value dictated by the initial random fluctuations, quantum noise makes θ diffuse; and (iii) the phase quadrature of the mode $\mathbf{e}_d = -i(\mathbf{e}_x e^{-i\theta} - \mathbf{e}_y e^{i\theta})/\sqrt{2}$ is perfectly squeezed (we shall call this the *dark mode* in the following, as its polarization is orthogonal to the bright mode, and, hence, it is empty at the classical level).

However, Type II OPOs degenerated in frequency and polarization invariant do not seem to exist. Normally the signal modes have different frequencies (that difference, however, can be as small as 150 kHz [11]), and the only way by which the amplification can be made frequency degenerate is, as far as we know, by breaking the polarization symmetry [12,13]: a birefringent plate is introduced within the cavity, which couples the two orthogonally polarized signal modes, thus forcing their frequency degeneracy but fixing the phase difference between them and breaking the system polarization symmetry. Hence, given the difficulties of having frequency-degenerate Type II OPOs, we pass to consider an alternative.

We propose a $\chi^{(3)}$ cavity in which SPSB can squeeze the dark output mode. Consider an isotropic $\chi^{(3)}$ medium placed inside a polarization isotropic cavity, pumped by two copropagating orthogonally polarized modes of frequencies ω_1 and ω_2, such that $\omega_s = (\omega_1 + \omega_2)/2$ is close to a cavity mode resonance ω_c. Unlike in OPOs, operation in frequency degeneracy has been experimentally proved in this kind of system [14], i.e., the pump modes can be mixed to generate a signal field with frequency ω_s via four-wave mixing. For the sake of simplicity, we shall treat the pumping modes as classical fields and will further ignore their depletion in interacting with the intracavity signal modes.

We write the total field at the cavity waist plane (where the $\chi^{(3)}$ medium is placed) as $\hat{\mathbf{E}}(\mathbf{r},t) = \mathbf{E}_p(\mathbf{r},t) + \hat{\mathbf{E}}_s(\mathbf{r},t)$:

$$\mathbf{E}_p = i\mathcal{F}G(\mathbf{r})\sum_{j=1,2}(\mathbf{e}_x\alpha_{jx} + \mathbf{e}_y\alpha_{jy})e^{-i\omega_j t} + \text{c.c}, \tag{2a}$$

$$\hat{\mathbf{E}}_s = i\mathcal{F}G(\mathbf{r})[\mathbf{e}_x\hat{a}_x(t) + \mathbf{e}_y\hat{a}_y(t)]e^{-i\omega_s t} + \text{H.c.}, \tag{2b}$$

where $\mathcal{F}^2 = \hbar\omega_s/(2\varepsilon_0 nL)$ (n is the refractive index and L the effective cavity length), and $G(\mathbf{r}) = \sqrt{2/\pi}w^{-1}\exp(-r^2/w^2)$ is the cavity TEM_{00} mode with radius w (which, for simplicity, we assume to be the same for the pump and signal modes, as their frequencies are assumed to be similar). Using the properties of the nonlinear susceptibility of isotropic media and ignoring its dispersion in the working frequency range [15], the interaction picture Hamiltonian describing our $\chi^{(3)}$ cavity reads $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$, where

$$\hat{H}_0 = \hbar\delta(\hat{a}_x^\dagger\hat{a}_x + \hat{a}_y^\dagger\hat{a}_y), \tag{3a}$$

$$\hat{H}_{\text{int}} = \frac{3}{4}\hbar g(\hat{H}_{\text{spm}} + \hat{H}_{\text{cpm}} + \hat{H}_{\text{fwm}}), \tag{3b}$$

with $\delta = \omega_c - \omega_s$ and $g = -8\varepsilon_0 l\chi_{xxxx}\mathcal{F}^4/(\hbar\pi w^2)$. We assumed that the $\chi^{(3)}$ medium length l is smaller than the cavity Rayleigh length. In $\hat{H}_{\text{int}}$, the terms $\hat{H}_{\text{spm}}$, $\hat{H}_{\text{cpm}}$, and $\hat{H}_{\text{fwm}}$ describe self-phase modulation, cross-phase modulation, and four-wave-mixing processes, respectively, and read

$$\hat{H}_{\text{spm}} = \hat{a}_x^{\dagger 2}\hat{a}_x^2 + \hat{a}_y^{\dagger 2}\hat{a}_y^2, \tag{4a}$$

$$\hat{H}_{\text{cpm}} = \sum_{j=1,2}4(|\alpha_{jx}|^2 + \mathcal{A}|\alpha_{jy}|^2)\hat{a}_x^\dagger\hat{a}_x + \sum_{j=1,2}4(|\alpha_{jy}|^2 + \mathcal{A}|\alpha_{jx}|^2)\hat{a}_y^\dagger\hat{a}_y + 4\mathcal{A}\hat{a}_x^\dagger\hat{a}_x\hat{a}_y^\dagger\hat{a}_y, \tag{4b}$$

$$\begin{aligned}\hat{H}_{\text{fwm}} = {} & \mathcal{B}\hat{a}_x^{\dagger 2}\hat{a}_y^2 + 2(\alpha_{1x}\alpha_{2x} + \mathcal{B}\alpha_{1y}\alpha_{2y})\hat{a}_x^{\dagger 2} \\ & + 2(\alpha_{1y}\alpha_{2y} + \mathcal{B}\alpha_{1x}\alpha_{2x})\hat{a}_y^{\dagger 2} \\ & + \sum_{j=1,2}4(\mathcal{B}\alpha_{jx}^*\alpha_{jy} + \mathcal{A}\alpha_{jx}\alpha_{jy}^*)\hat{a}_x^\dagger\hat{a}_y \\ & + 4\mathcal{A}(\alpha_{1x}\alpha_{2y} + \alpha_{1y}\alpha_{2x})\hat{a}_x^\dagger\hat{a}_y^\dagger + \text{H.c.},\end{aligned} \tag{4c}$$

with $\mathcal{A} = \chi_{xxyy}/\chi_{xxxx}$ and $\mathcal{B} = \chi_{xyyx}/\chi_{xxxx}$, which verify $2\mathcal{A} + \mathcal{B} = 1$ [8].

To preserve polarization invariance, the classical pumping fields must necessarily have orthogonal circular polarizations, as any other polarization would privilege particular spatial directions in the transverse plane. Consequently, we take $\alpha_{1x} = \alpha_{2x} = \rho/\sqrt{2}$ and $\alpha_{1y} = -\alpha_{2y} = i\rho/\sqrt{2}$. Then we rewrite the Hamiltonian in the basis of the circularly polarized states $\hat{a}_\pm = (\hat{a}_x \mp i\hat{a}_y)/\sqrt{2}$ and get

$$\hat{H}_0 = \hbar\delta(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-), \tag{5a}$$

$$\hat{H}_{\text{spm}} = (1 - \mathcal{B})(\hat{a}_+^{\dagger 2}\hat{a}_+^2 + \hat{a}_-^{\dagger 2}\hat{a}_-^2), \tag{5b}$$

$$\hat{H}_{\text{cpm}} = 2(1 + \mathcal{B})\hat{a}_+^\dagger\hat{a}_+\hat{a}_-^\dagger\hat{a}_- + 2\rho^2(3 - \mathcal{B})(\hat{a}_+^\dagger\hat{a}_+ + \hat{a}_-^\dagger\hat{a}_-), \tag{5c}$$

$$\hat{H}_{\text{fwm}} = 2\rho^2(1 + \mathcal{B})(\hat{a}_+\hat{a}_- + \hat{a}_+^\dagger\hat{a}_-^\dagger). \tag{5d}$$

Just as $\hat{H}_{\text{OPO}}$, this Hamiltonian has the symmetry $(\hat{a}_+, \hat{a}_-) \to (\hat{a}_+ e^{i\theta}, \hat{a}_- e^{-i\theta})$ with $\hat{a}_+^\dagger\hat{a}_+ - \hat{a}_-^\dagger\hat{a}_-$ as the associated constant of motion. Hence, whenever a nonzero mean field appears for the signal modes, bright emission will take place in the mode $\mathbf{e}_b = (\mathbf{e}_+ e^{-i\theta} + \mathbf{e}_- e^{i\theta})/\sqrt{2} \equiv \mathbf{e}_\theta$, i.e., the mean field will have a linear polarization along the arbitrary θ axis [7] ($\mathbf{e}_\pm$ are the right and left circularly polarized modes), thus breaking the polarization symmetry of the system. Then quantum fluctuations should induce a diffusion process in the FPP θ, allowing for perfect noncritical squeezing in a quadrature of the dark mode $\mathbf{e}_d = -i(\mathbf{e}_+ e^{-i\theta} - \mathbf{e}_- e^{i\theta})/\sqrt{2} \equiv \mathbf{e}_{\theta+\pi/2}$, which is crossed polarized with respect to the bright mode. Note that these linearly polarized modes are mapped onto the elliptically polarized bright and dark modes of the Type II OPO by a quarter-wave plate [7].

Fortunately, it will not be necessary to prove the above conclusions explicitly, as by taking $\mathcal{A}=\mathcal{B}=1/3$ (which applies when the Kleinman symmetry is verified, such as in nonresonant electronic response [8]) Hamiltonian (5) becomes isomorphic to that of [3], where we already proved that for $\delta>\sqrt{3}\gamma_s$ and for pump intensities ρ^2 inside the region defined by the curves $\rho^2=\gamma_s/2$ g and $\rho^2=(2\delta+\sqrt{\delta^2-3\gamma_s^2})/6$ g, a steady, nonzero mean field solution for the signal modes appears through a subcritical pitchfork bifurcation. The perfect and noncritical squeezing of a quadrature of the dark mode was also proved within the linear approximation for quantum fluctuations. Except for the quantitative details, these results hold for $\mathcal{A}\neq\mathcal{B}$.

Some comments are in order here. First, one may think that the undepleted pump approximation is too drastic. Pump depletion could introduce new bifurcations in the system, limiting the stability region of the cw mean field. However, as the only relevant ingredient needed to generate the aforementioned properties of the signal field is SPSB, these would remain unaltered within the stability region of the cw solution. Second, in [2] we demonstrated for the two-transverse-mode DOPO by direct numerical integration of the system quantum dynamic equations, both the random rotation of the bright mode and the perfect noncritical squeezing of the dark mode, hence showing that these properties hold beyond the linear approximation. Finally, in [1] we proved that small deviations from perfect rotational symmetry do not imply a large degradation of the dark mode quadrature squeezing [16], while it can fix the bright and dark mode orientation, which seems quite advantageous from the experimental point of view in order to facilitate the homodyning of the dark mode. These conclusions will doubtless apply to the $\chi^{(3)}$ cavity we have presented here.

Finally, note that although one of the Stokes parameters is free from quantum fluctuations in the output light (the twin beam intensity difference), this is not enough to claim for polarization squeezing, as further conditions must be satisfied [17,18]. In any case, study of the polarization squeezing is out of the scope of this Letter.

In conclusion, we have theoretically demonstrated that Type II frequency-degenerate OPOs and appropriate $\chi^{(3)}$ cavities are suitable for the generation of noncritical and perfect quadrature squeezing. We have commented on the problems that actual Type II OPOs have (either they are not exactly frequency degenerated or have a cavity that is not polarization symmetric). We believe that the $\chi^{(3)}$ cavity we are proposing does not present these problems, and it can be built within the experimental state of the art, hence being a good candidate for observing SPSB.

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